Al-Saudia Virtual Academy
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Limits

If f is a function, then we say that $\lim_{x->a.} f(x) = f(a)$

If the value of f (x) gets arbitrarily close to f(a) as x gets arbitrary close to a.

Theorem:- Let lim stands for one of the limit

$\lim_{x->a.}$, $\lim_{x->a-.}$, $\lim_{x->a+.}$, $\lim_{x->0.}$, $\lim_{x->0-.}$, $\lim_{x->0+.}$

and If L_1 = lim f (x) L_2 = lim g (x) both exist, then.

(a) Lim [f (x) + g (x)] = lim f (x) + lim g (x) = $L_1 + L_2$

(b) Lim [f (x)] = lim f (x) – lim g (x) = $L_1 - L_2$

(c) Lim [f (x) g (x)] = lim f (x) = $L_1 . L_2$

(d) $\frac{\lim f(x)}{\lim (gx)}$ = L_1/L_2 where $L_2 \neq 0$

(e) Lim n$\sqrt{f(x)}$ =$n\sqrt{\lim f(x)}$ = $n\sqrt{L_1}$ Provide $L_1 > 0$ if n is even.

In words, these theorems states:

(a) The limit of a sum is the sum of the limits.

(b) The limit of a difference is the difference of the limits.

(c) The limit of a product is the product of the limits.

(d) The limit of a quotient is the quotient of the limits.

Provided the limit of the denominator is non zero.

(e) The limit of an nth roof is the nth not of the limit.

THEOREM: For any polynomial

$$P (x) = Co + c, x + ... + c_n x_n$$

and any real number a,

$$\lim_{x\to a} p(x) = c_o + c_1 + ... + c_n a_n = p(a)$$

There is an important principal about limits of polynomials which expressed informally states that a polynomial behaves like its term of highest degree as $x \to \infty^{+}$ or $x \to \infty^{-}$More precisely, if c_n# 0, then

$$\lim_{x\to\infty} c_0 + c_1 + \cdots c_n a_n = \lim_{x\to\infty} c_n x_n$$

A QUICK METHOD FOR FINDING LIMITS OF RATIONAL FUNCTIONS

$$\lim_{x\to\infty} C_o + C_1 + \ \ldots\ldots\ldots.. + C_n X_n = \lim_{x\to\infty} C_n X_n$$

$$d_{o\,+}\, d_1 + \ldots\ldots. + d_m x_m \quad x \to \infty\, d_m x_m$$

LIMITS OF PIECE – WISE DEFINED FUNCTIONS:-

For functions that are defined piecewise, a two – sided limit at a point where the formula for the function changes is best obtained by first finding the one – sided limit at the point.

IMPORTANT LIMITS:-

The following limits are important

(1) $\lim_{x\to 0} sinx = 1$
(2) $\lim_{x\to 0} cosx = 0$
(3) $\lim_{x\to 1} \frac{x_n - 1}{x-1} = n$
(4) $\lim_{x\to\infty} (1 + \frac{1}{x})^{x} = e$
(5) $\lim_{x\to 10} \frac{a^{x}-1}{x} = \log_e a$

To prove above theorem (1) we first understand squeeze theorem.

THEOREM: (The squeezing theorem) :- let f, g, and h be the functions satisfying g(x) $\leq$f(x) $\leq$ h(x) for all x in some open interval containing the point a, with the possible exception that the in equalities need as x approaches a, say

$$\lim_{x\to a} g(x) = \lim_{x\to a} f(x) = L$$

Then f also has this limit as x approaches a, that is, $\lim_{x\to a} f(x) = L$

Example: Use the squeeze theorem to evaluate the limit $\lim_{x\to 0} x^2 \sin^2(\frac{1}{x})$

Solution . If x $\neq$ o , we can write

$$O \leq \sin^2(\frac{1}{x}) \leq 1$$

Multiplying through by x^2 yields.

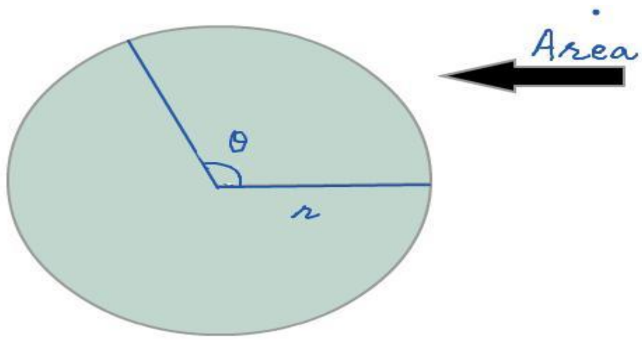
$$O \leq x^2 \sin^2(\frac{1}{x}) \leq 1$$

But $\lim_{x\to 0}$ 0= $\lim_{x\to 0} x^2$=0 , so by the squeezing theorem

$$\lim_{x\to 0} x^2 \sin^2\left(\frac{1}{x}\right) = 0$$

AREA OF A SECTOR “A” of a circle with radius r and central angle θ radius.

$$\frac{A}{\pi r^2} = \frac{\theta}{2\pi}, \left(\frac{\text{area of the sector}}{\text{area of the circle}} = \frac{\text{central angle of the sector}}{\text{central angle of the circle}}\right)$$



from which we obtain the formula.

$$A = ½ r^2\theta$$

PROVE:-

$$\lim_{h\to 0}\frac{sinh}{h} = 1$$

Proof:- Let a circle with radius

V = 1 and assume that $0 \le h \le \pi/2$, and construct the angle h in standard position. The terminal side of "h" intersect the unit circle at P(cos h, sin h) and draw a tangent at B, extend OP to tangent line meet at Q (1, tan h). From figure we obtain.

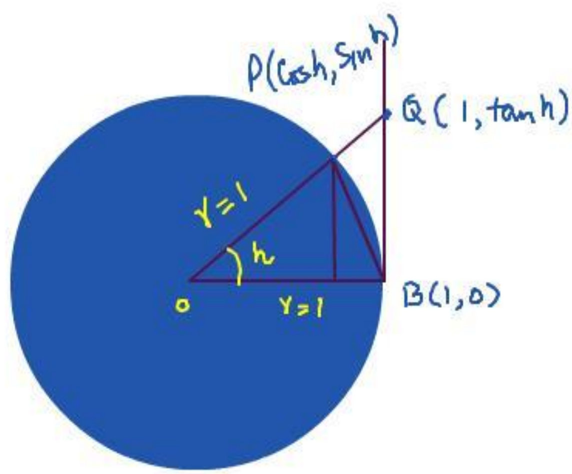
O < area of △ OBP < area of sector OBP
< area of △ OBQ. ……. (i)

But.

Area of △ OBP = ½ . base . altitude.

= ½ . (1) . sin h = ½ sin h………..(ii)

And area of sector OBP = A = ½ $r^2\,\theta$.



$=\frac{1}{2}.(1)^2.h \ = \ \frac{h}{2}$ …………………… (iii)

Also area of △ OBQ = 1/2. base. altitude.

= ½ .(I). tan h

$= \frac{tanh}{2}$ ………………… (4)

Therefore

½ sin h < h/2 < $\frac{tan\,h}{2}$

Multiplying through by 2/sin h yields.

$$I \ < \ \frac{h}{\text{Sin h}} < \ \frac{1}{cosh}$$

Taking reciprocal

Cos h > sin h > I ……………. (5)

We have derived (5) under assumption 0< h < π/2, however, this inequality is also valid if (-π/2< h < 0), so that it is true for all "h" in the interval (-π/2, π/2) except h = o, but

$$\lim_{h\to 0} Cos\ h = 1\ \ and\ \ \lim_{h\to 0} 1 = 1$$

So by squeezing theorem

$$\lim_{h\to 0}\frac{\sin h}{h} = \ 1$$
