Circular Motion and Gravitation

Circular Motion:

When an object revolves in a circular orbit its motion is said to be circular motion.

Uniform Circular Motion:

When an object revolves in a circular orbit with uniform speed its motion is said to be uniform circular motion. OR

When a body moves in a circular orbit in such a way that it covers equals distance in equal interval of time them its motion is said to be uniform circular motion.

examples of uniform circular motion are as follows:

(1) Motion of electron around the Nucleus.

(2) Motion of planet around the sun.

(3) Any object which is moving in closed circular orbit.

Centripetal Acceleration:

When an object revolves in a circular orbit with uniform speed its motion is said to be uniform circular motion. During uniform circular motion magnitude of velocity remains same but the direction which is always tangent to the circle changes at every point. Such acceleration which is due to the change in direction not because of magnitude is called centripetal acceleration. The direction of centripetal acceleration is towards centre. OR

A moving object in a circular orbit with uniform speed possesses an acceleration which always directed towards centre. Such Acceleration is called centripetal acceleration. It is denoted by $\mathbf{a_c}$.

Mathematically centripetal acceleration is expressed as

$$\mathbf{a_c} = \frac{\mathbf{V^2}}{\mathbf{r}}$$

Where $\mathbf{a_c}$ = centripetal acceleration, V^2 = speed of the moving object, r = radius of the circular orbit. Its unit is m/s^2.

Centripetal Force:

"The force which is responsible for the motion of a body in a circular orbit is called centripetal force." OR "The force which keeps the body revolving in a circular orbit is called centripetal force." The direction of centripetal force is always towards the centre. According to Newton's 2nd law of motion, the force responsible for the motion of a body is given as

$$\mathbf{F = ma} \text{ \_\_\_\_\_\_\_\_\_\_ (i)}$$

For centripetal Force, having direction towards center.

$$\mathbf{F_c = m\,a_c} \text{ \_\_\_\_\_\_\_\_\_\_ (ii)}$$

Where F_c is the centripetal force and ac is the centripetal acceleration.

As we know that centripetal acceleration is given as

$$\mathbf{a_c = \frac{V^2}{r}} \text{ \_\_\_\_\_\_\_\_\_\_ (iii)}$$

Substituting (iii) in (ii)

$$\mathbf{F_c = \frac{mV^2}{r}}$$

This is the required expression of centripetal force.

Examples:

1. A satellite Circling the earth, the necessary force is supplied by gravity which is directed towards the centre of the earth such force is called the centripetal force.
2. The planets revolve round the Sun; the centripetal force is then due to the gravitational force.
3. A ball of mass 'm'is attached to the one end of the string. The ball is whirled by moving the hand in a circle in a horizontal direction. The ball traveling in a circle is under the influence of centripetal force which is directed towards the center of the circle.

Centrifugal Force:

When an object revolves in a circular orbit it is under the influence of centripetal force which is directed towards centre. This force acts as an action. According to Newton's 3rd Law of motion a reaction generates which has the same direction as that of centripetal force but acts always in opposite direction i.e., away from the centre. This reacting force is called centrifugal force. OR "The force which is the reaction of centripetal force is called centrifugal force. The magnitude of centripetal and centrifugal force is just exactly same and their directions are just opposite."

Action Force = Centripetal Force

Reaction Force = Centrifugal Force.

According to Newton's 3rd Law of motion

Action Force = -Reaction Force

Where negative sign shows the opposite direction.

Centripetal Force = Centripetal Force

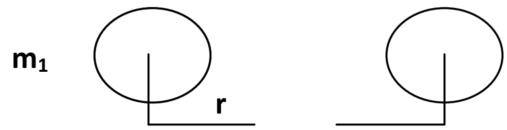
Centripetal Force = $\mathbf{F_c = \frac{mV^2}{r}}$

Force of Gravitation:

The word gravitation is associated with attraction. The force with which body attracts any other body is called gravitational force. The gravitational force effect on anybody depends on its density. As the density of the object increases its gravitation effect i.e. gravitational attractive force increases. The gravitational effect of body also depends on the distance. As the test object moves away from the source the gravitational force between then decreases.

Newton's law of gravitation:

Statement:



Newton's law of gravitation stated as: "every two bodies in this universe attracts each other with a force which is directly proportional to the product of their masses and Inversely proportional to the square of the distance between them."

Explanation of mathematical expression:

Newton's law of gravitation consists of two parts.

According to first part gravitational force is directly proportional to the product of their masses.

If m1 and m2 are the masses of bodies A and B respectively, then according to the law,

$$\mathbf{F} \infty \mathbf{m_1 m_2} \quad \_\_\_\_\_\_\_\_\_\_ (1)$$

Above expression shows that if the objects are huge then gravitational force between them will be change and the object will light then gravitational force will be weak.

According to the second part gravitational force is inversely proportional to the square of the distance between centers.

If the **"r "**is the distance between the centre of both objects then according to the law.

$$\mathbf{F} = \frac{\mathbf{1}}{\mathbf{r^2}} \quad \_\_\_\_\_\_\_\_\_\_ (2)$$

Combining the equations (1) and (2)

$$\mathbf{F} = \frac{\mathbf{m_1 m_1}}{\mathbf{r^2}} \_\_\_\_\_\_\_\_\_\_\_\_\_\_ (3)$$

Removing the proportional sign:

$$\mathbf{F} = \mathbf{G}\frac{\mathbf{m_1 m_1}}{\mathbf{r^2}}$$

Where G is the proportionality constant and is called universal gravitational constant.

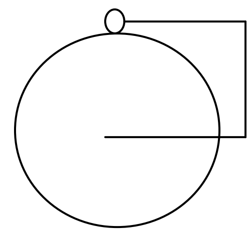
In S.I. System the value of G is **6.67 x 10-11 N-m^2 / kg^2**

Mass of earth:

Suppose a body of mass "m"is placed on the surface of earth. The distance between the centres of the body to the centre of the earth is R as show in figure. R_E

According to Newton's law of Gravitation.

$$\mathbf{F = G\frac{m_1 m_1}{r^2}}$$



From figure $m_1 = M_E$, $m_2 = m$, $r = R$

$$\mathbf{F = G\frac{M_E m}{R^2}}$$

But R = R_E + r b

$$\mathbf{F = G\frac{M_E m}{(R_E + r_b)^2}}$$

$$\mathbf{r_b <<< R_E}$$

$\mathbf{r_b}$ is so small as compared to R_E that it can be neglected.

$$\mathbf{r_b = 0}$$

This is the force with which earth attracts the body toward its centre and by definition it is equal to the weight of the body.

$$\mathbf{F = W}$$

$$\mathbf{W = G\frac{M_E m}{{R_E}^2}}$$

$$\mathbf{mg = G\frac{M_E m}{{R_E}^2}}$$

$$\mathbf{g = G\frac{M_E}{{R_E}^2}}$$

$$\mathbf{M_E = \frac{g{R_E}^2}{G}}$$

This is the required expression for the mass of earth. As we have the values:

g = 9.8 m /s2

R_E = 6.4 x 10 6m

G = 6.67 x 10-11 N – m2 / kg2

On subtitling values we get: **$M_E = 5.98 \times 10^{24}$ kg**

Variation of "g" with altitude:

Using expression for the mass of earth

$$\mathbf{M_E = \frac{gR_E{}^2}{G}}$$

$$\mathbf{g = \frac{M_E G}{R_E{}^2}}$$

$$\mathbf{g \infty 1/R^2}$$

Above expression shows that "g" depends inversely on "R" which is the distance from the centre of the earth to the centre of body. Therefore the value of "g" decreases and R increases. It means if object goes to height the g acting on it decreases.